

# C-Graph

---

A Tool for Learning about Convolution  
C-Graph version 2.0, 16 March 2011

Adrienne Gaye Thompson ([agt@codeartnow.com](mailto:agt@codeartnow.com))

---

This is a manual for C-Graph version 2.0, a tool for learning about convolution.

Copyright © 2011 Free Software Foundation, Inc.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, with no Front-Cover Texts, and with no Back-Cover Texts. A copy of the license is included in the section entitled “GNU Free Documentation License”.

for **ALL** the **VICTIMS** of **APARTHEID** **STRUGGLING** to be **FREE**

*and*

to **REGNIER**

*You're sending me discrete signals from across the room,*

*I respond on impulse, reflecting on the sampling of events*

*That were a dichotomy from the day you left your mother's womb;*

*Multiplied in frequency, integrated in time, a weighted confluence*

*Of sliding, shifting trains of thought, alternative messages under transformation;*

*Counterpoint, duality, involution, contradistinction without confusion;*

*Independence in summation. Silence - this is convoluted conversation.*



# Table of Contents

<b>1</b>	<b>Foreword</b> .....	<b>3</b>
<b>2</b>	<b>Overview</b> .....	<b>5</b>
2.1	About .....	5
2.2	Required Software.....	5
<b>3</b>	<b>Invoking C-Graph</b> .....	<b>7</b>
<b>4</b>	<b>The Signals</b> .....	<b>9</b>
4.1	The Menu.....	9
4.2	Parameters.....	9
4.3	Defaults & Error Handling .....	9
4.4	Frequency Selection .....	10
<b>5</b>	<b>A C-Graph Session</b> .....	<b>13</b>
<b>6</b>	<b>Reporting Bugs</b> .....	<b>25</b>
<b>Appendix A</b>	<b>Sketch of Convolution Theory</b> ...	<b>27</b>
A.1	Introductory Ideas.....	27
A.2	The Mathematics.....	27
A.2.1	Deriving the Convolution Sum .....	27
A.2.2	The Convolution Theorem .....	28
A.3	Linear and Circular Convolution.....	29
A.3.1	Linear Convolution .....	29
A.3.2	Circular Convolution .....	31
<b>Appendix B</b>	<b>References</b> .....	<b>35</b>
<b>Appendix C</b>	<b>Signals Gallery</b> .....	<b>37</b>
<b>Appendix D</b>	<b>GNU Free Documentation License</b> .....	<b>39</b>
<b>Index</b> .....		<b>47</b>



# 1 Foreword

From the shadow cast by light to the echo in a cave convolution, like the ubiquitous Fibonacci series, is a mathematical description of naturally occurring physical phenomena in any linear, time-invariant system capable of responding to an input signal. Today, convolution - the combination of two signals to produce a third - has wide ranging applications. Edge detection in computer vision, algorithms for robot motion, signal and image processing, crystallography, statistics and probability theory, differential equations, linear algebra, numerical analysis, and even recent innovations in music production - all utilise techniques involving convolution.

C-Graph (for Convolution Graph) is a tool for visualizing the convolution of two signals. The package is, essentially, a rewrite of the Fortran 77 program in my BSc. Electrical Engineering (Honours) dissertation "Interactive Computer Program Demonstrating: Sampling Convolution and the FFT", University of Aberdeen, Scotland, 1983. In this version I have included pulses, scaling of the signals, and error-handling - features that were not part of my original Thesis.

Whether student engineer or scientist, aspiring audio producer or image analyst, C-Graph will help you find the adventure in the mathematics of convolution.

– *Adrienne Gaye Thompson*





## 2 Overview

### 2.1 About

C-Graph computes the linear convolution of two signals in the time domain then compares their circular convolution by demonstrating the *convolution theorem* - convolution of two signals in the time domain corresponds to multiplication in the frequency domain. Each signal is modelled by a register of discrete values simulating samples of a signal, and the discrete Fourier transform (DFT) computed by means of the Fast Fourier Transform (*FFT*). See [Appendix A \[Appendix A\], page 27](#), for an explanation of the convolution theorem.

C-Graph is interactive, prompting the user to enter single character or numerical values from the keyboard, thereby dispensing with the learning curve for coding formulae. The user chooses 2 from a menu of 8 signal types, and up to 5 parameters to define the waveforms. The signals chosen may be periodic, aperiodic, or pulses. C-Graph then plots 3 graphs:

1. The time domain representation of both signals;
2. Their Fourier transforms;
3. A comparison of their linear and circular convolution.

See [Chapter 5 \[A C-Graph Session\], page 13](#), for a typical C-Graph session.

C-Graph will be useful to students of signal theory in the study of convolution and spectral analysis. This version (2.0) uses a simple FFT written by Arthur Wouk and converted to Fortran 90 by Alan Miller (see [Appendix B \[Appendix B\], page 35](#)).

### 2.2 Required Software

C-Graph is written in modern Fortran. The package runs on GNU/Linux, was developed with GFortran and G95, and uses Gnuplot 4.2 as well as Image Magick 6.6.

Experienced users wishing to use other compilers may supply the necessary command-line options to `configure` during installation. See the file 'INSTALL' for basic installation instructions.



## 3 Invoking C-Graph

To run C-Graph, open up a terminal in X and type `c-graph`. C-Graph supports the following options:

`--help`

`-h` Print a summary of the command line options and exit.

`--version`

`-v` Print the version number and licensing information of C-Graph, then exit.

`--dedicate`

`-d` Print the dedication and exit.



## 4 The Signals

### 4.1 The Menu

C-Graph presents the following menu of signals from which the user chooses 2:

SIGNAL	CODE
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Exponential	F
Ramp	G
Step	H

Signals ‘A’ to ‘E’ are periodic, while ‘F’, ‘G’, and ‘H’ are aperiodic. Pulses may also be chosen; these are a half period in duration  $1/2f$ , where  $f$  is the frequency of the corresponding cyclical waveform.

### 4.2 Parameters

The user enters up to 5 parameters to generate the signals, their FFTs and their convolution:

1. The number of samples ‘N’
2. The code for the signal ‘A’ to ‘H’
3. The wave/pulse parameter ‘w’ or ‘p’
4. The frequency ‘f’
5. The scaling coefficient ‘sc’

Both signals are constructed from the same number of samples ‘N’. If the user chooses a periodic signal, then he/she is prompted to select either the cyclical waveform or a derived pulse, i.e., ‘w’ or ‘p’. For each periodic signal chosen the user is prompted to enter its frequency ‘f’ and a scale factor ‘sc’.

Pulses are monophasic and are defined on half the period of the modulus of the corresponding periodic waveform.

### 4.3 Defaults & Error Handling

If the required parameter is a number and the user has erroneously entered character data, C-Graph generates an error message and gives the user another try to enter a number. Otherwise, for input outside the expected ranges c-graph assumes default values.

Number of samples ‘N’

‘N’ must lie in the range [64, 1024]. Values entered outside of this range will default to 512. ‘N’ is defined to be a power of 2. If the user enters a value that is not a power of 2 C-Graph will choose the nearest power of 2.

**Signal code 'A' to 'H'**

For input outside the range 'A' to 'H', the default codes are 'C' for the first signal, and 'D' for the second.

**Wave/Pulse parameter 'w' or 'p'**

The default waveform is a pulse.

**Frequency 'f'**

C-Graph assumes a default frequency of 1Hz for values of 'f' entered outside the range  $[0.5, N/4]$ .

**Scaling coefficient 'sc'**

The scaling coefficient 'sc' may be positive or negative. The maximum absolute value of 'sc' for signals a, b, f, and h is 'N', while that for signals c, d, e, and g is 1. All signals will be scaled to unity for input values of 'sc' outside the permitted range.

With the default scaling coefficient of 1, signals a, b, f, and h are unit functions; signals d (square) and e (sawtooth) have a maximum amplitude of half the period ( $1/(2f)$ ) while that of c (triangle) is one-quarter the period ( $1/(4f)$ ).

## 4.4 Frequency Selection

We can express the period  $P$  of a periodic signal as

$$\begin{aligned} P &= N/(\text{number of cycles}) \\ &= T/(N/n) \end{aligned}$$

where  $T$  is the duration of the signal register in seconds,  $N$  is the number of samples in the register (window length), and  $n$  is the number of samples in 1 period.

The frequency of the signal  $f$  is the reciprocal of the period, so

$$f = N/(nT) \text{ samples/seconds or Hz}$$

C=Graph assumes that the duration of the signal register is 1 second, so

$$f = N/n \text{ Hz}$$

The sampling rate  $f_s$  is given by

$$f_s = N/T \text{ Hz}$$

The interval  $h$  between successive samples being the reciprocal

$$h = 1/f_s \text{ seconds}$$

If the window length and frequency chosen are 512 and 20 Hz (approximately the lower limit of the human audible range) then the number of samples  $n$  in each period would therefore be

$$\begin{aligned}n &= N/f \\ &= 512/20 \\ &= 25.6\end{aligned}$$

C-Graph requires that  $n$  be a multiple of 4. For each periodic signal, the frequency entered by the user is accordingly adjusted so that  $n$  approximates to the nearest multiple of 4. So the frequency of the signal used by C-Graph would become

$$\begin{aligned}f &= N/n \\ &= 512/26 \\ &= 19.7 \text{ Hz}\end{aligned}$$





## 5 A C-Graph Session

In this session, we run C-Graph twice to compare the convolution of 2 signals of equal length:

1. When the signals are finite sequences and the remaining interval is zero padded to the ‘N’ with at least the same number of zeros as samples in each signal;
2. When both signals extend across the full register of ‘N’ samples.

We use a sawtooth pulse for the first signal, and a rectangular pulse half the amplitude of the sawtooth for the second signal. We also demonstrate the use of default values for unexpected input. The keychord *ALT-TAB* is used to toggle the terminal and the Gnuplot window.

In X, type `c-graph`. The following text will appear:

```
C-Graph version 2.0
Copyright © 2011 Free Software Foundation, Inc.
```

```
C-Graph is free software licensed under the terms of the
GNU General Public License (the GPL) version 3 or later.
```

```
C-Graph is distributed WITHOUT ANY WARRANTY
without even the implied warranty of MERCHANTABILITY
or FITNESS FOR A PARTICULAR PURPOSE.
```

```
See the GPL for more details.
```

```
Enter <g> to display the GPL, or <ENTER> to continue.
C-Graph:>> RET
```

```
This is C-Graph - a tool to visualize convolution.
```

```
Compare the linear convolution of two signals with their circular
convolution.
```

Signal	Code
=====	=====
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Unit-Exponential	F
Unit-Ramp	G
Unit-Step	H

Generate 2 signals from the above menu with up to 5 parameters:

1. Number of samples 'N'
2. Signal code 'A' to 'H'
3. Whether the signal, if periodic, is a wave 'w' or a pulse 'p'
4. The frequency 'f'
5. The scaling coefficient 'sc'

Choose a value for "N" between 64 and 1024.

C-Graph:>> 51

The number of samples "N" is: 512

Signal	Code
=====	=====
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Unit-Exponential	F
Unit-Ramp	G
Unit-Step	H

Enter code for first signal

C-Graph:>> e

Is the signal periodic or is it a pulse?

Type "w" for periodic wave, or "p" for pulse

C-Graph:>> p

Select the frequency "f" of this signal.

C-Graph:>> 1.0

The frequency of this signal is 1.00 Hz.

Do you wish to scale this signal?

Enter a value for the scaling coefficient "sc".

A coefficient of 1 will give the unit function.

C-Graph:>> 1

Signal	Code
=====	=====
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Unit-Exponential	F
Unit-Ramp	G
Unit-Step	H

Enter code for second signal

C-Graph:>> s

Is the signal periodic or is it a pulse?

Type "w" for periodic wave, or "p" for pulse

C-Graph:>> p

Select the frequency "f" of this signal.

C-Graph:>> 1

The frequency of this signal is 1.00 Hz.

Do you wish to scale this signal?

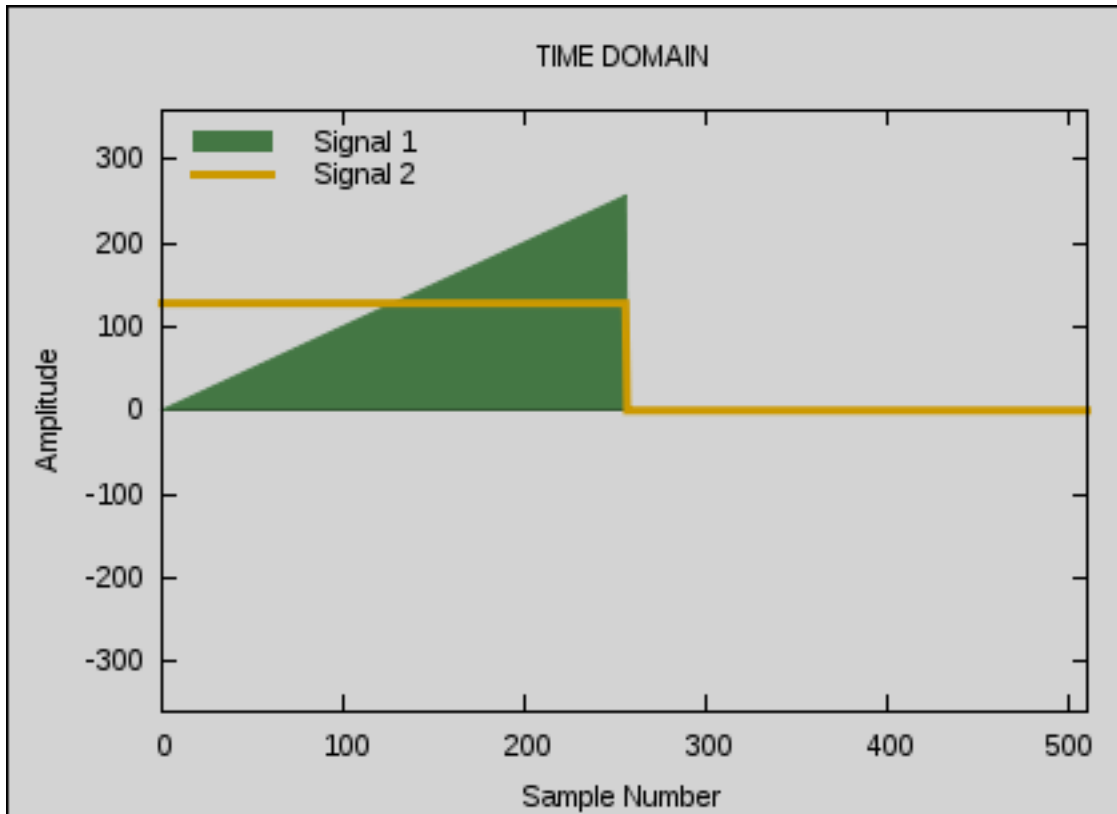
Enter a value for the scaling coefficient "sc".

A coefficient of 1 will give the unit function.

C-Graph:>> .5

You selected a square signal by default.

Press <Enter> to see the signals in the time domain:>> RET

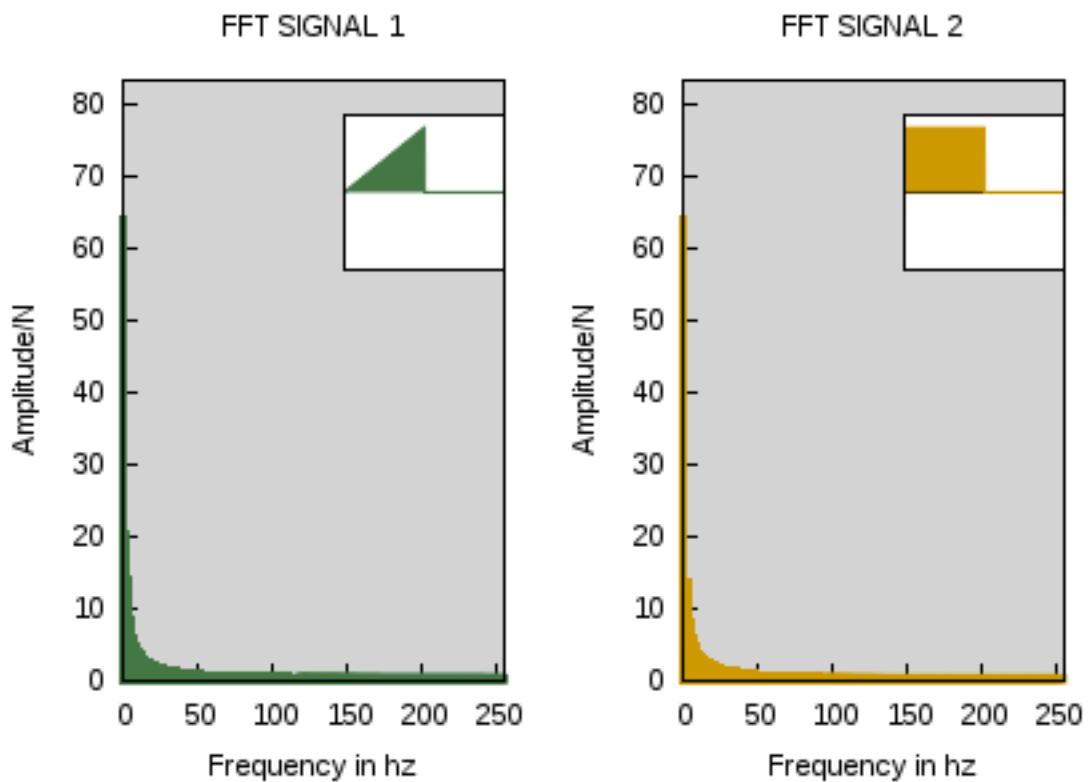


*ALT-TAB*

Hit <Enter> to continue:>> RET

View the frequency-domain representation of the signals.

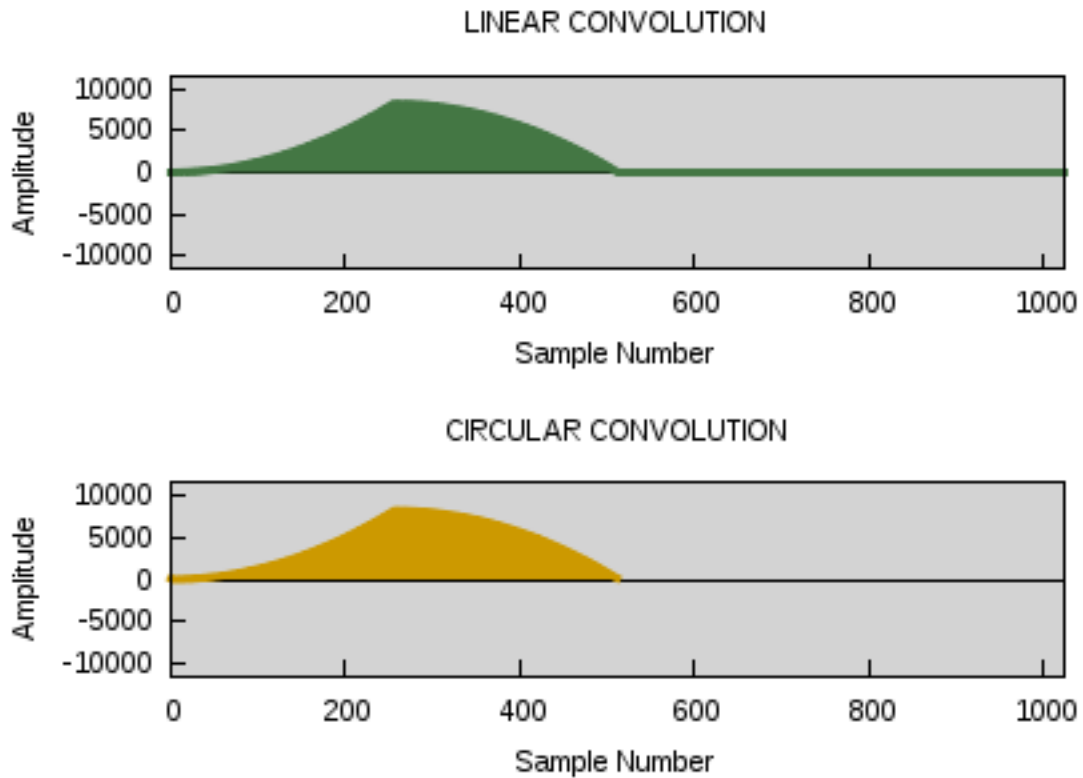
Press <Enter> to see their FFTs:>> RET



ALT-TAB

Hit <Enter> to continue:>> RET

Press <Enter> to compare linear and circular convolution:>> RET



*ALT-TAB*

Hit <Enter> to continue:>> RET

Run C-Graph again? Enter 'y' or 'n'.

C-Graph:>> y

This is C-Graph - a tool to visualize convolution.

Compare the linear convolution of two signals with their circular convolution.

Signal	Code
=====	=====
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Unit-Exponential	F
Unit-Ramp	G
Unit-Step	H

Generate 2 signals from the above menu with up to 5 parameters:

1. Number of samples 'N'
2. Signal code 'A' to 'H'
3. Whether the signal, if periodic, is a wave 'w' or a pulse 'p'
4. The frequency 'f'
5. The scaling coefficient 'sc'

Choose a value for "N" between 64 and 1024.

C-Graph:>> 512

The number of samples "N" is: 512.

Signal	Code
=====	=====
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Unit-Exponential	F
Unit-Ramp	G
Unit-Step	H

Enter code for first signal

C-Graph:>> g

Do you wish to scale this signal?

Enter a value for the scaling coefficient "sc".

A coefficient of 1 will give the unit function.

C-Graph:>> 1

Signal	Code
=====	=====
Sine	A
Cosine	B
Triangle	C
Square	D
Sawtooth	E
Unit-Exponential	F
Unit-Ramp	G
Unit-Step	H

Enter code for second signal

C-Graph:>> h

Do you wish to scale this signal?

Enter a value for the scaling coefficient "sc".

A coefficient of 1 will give the unit function.

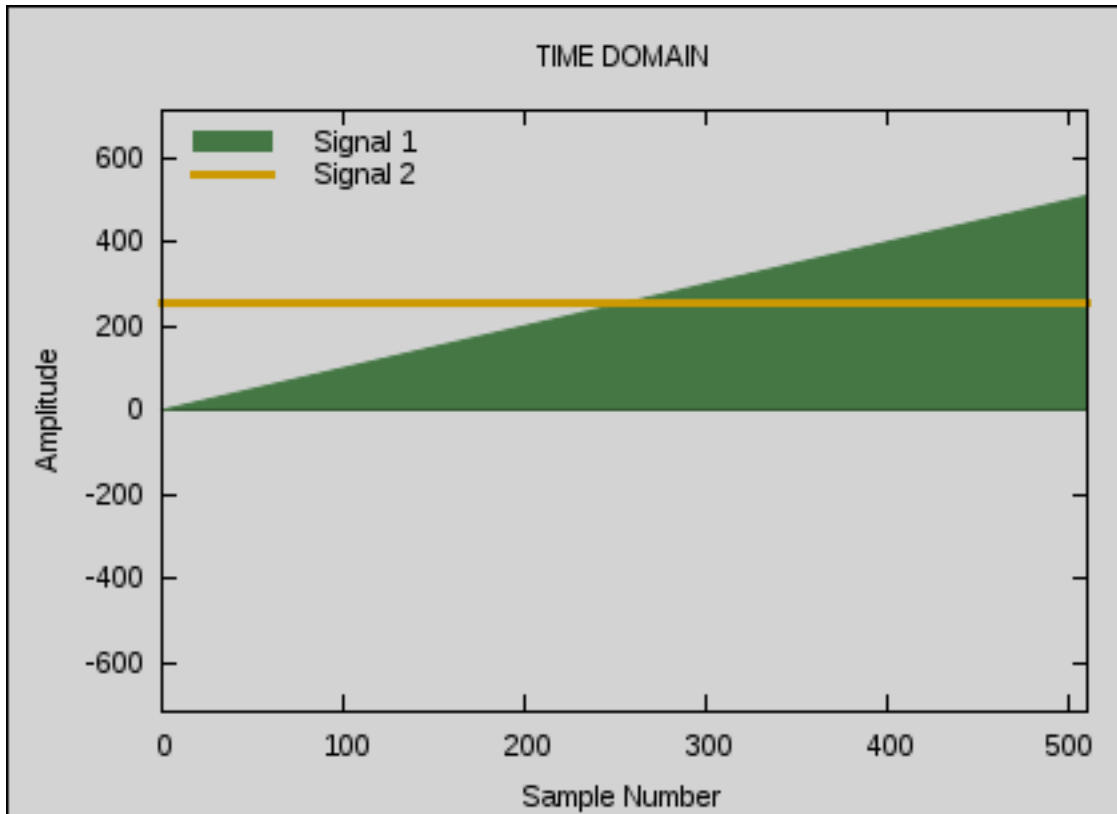
C-Graph:>> q56

That was not a number. Try again!

C-Graph:>> 256



Press <Enter> to see the signals in the time domain:>> RET

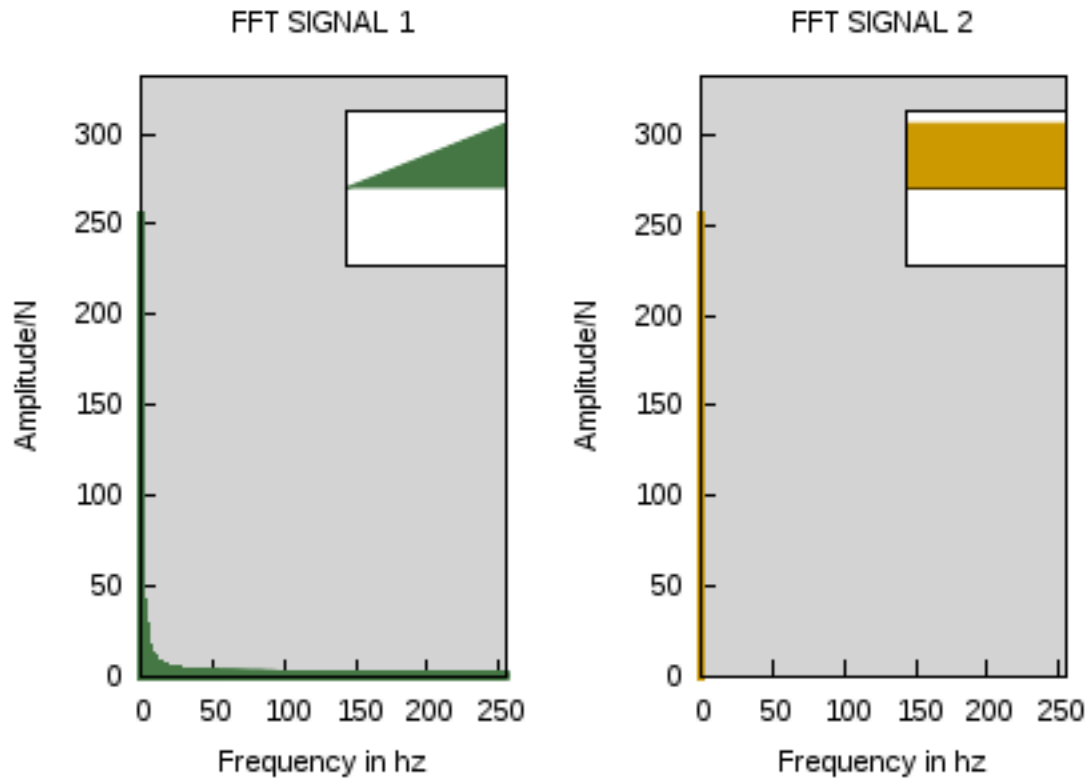


*ALT-TAB*

Hit <Enter> to continue:>> RET

View the frequency-domain representation of the signals.

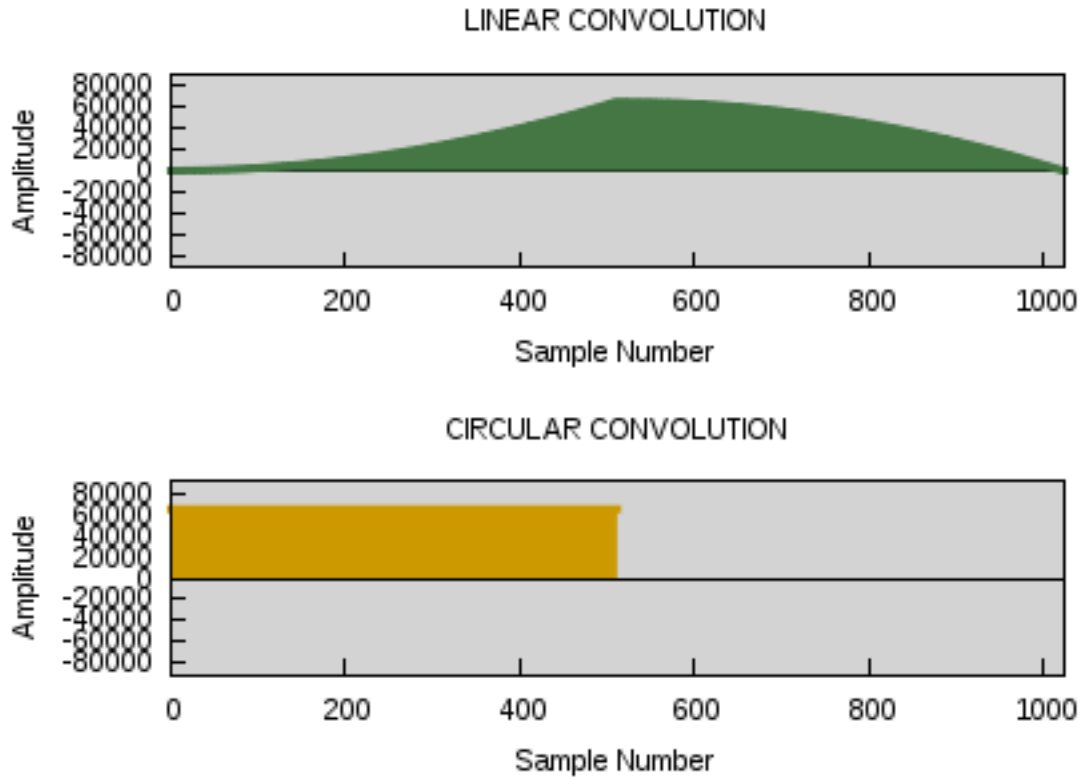
Press <Enter> to see their FFTs:>> RET



*ALT-TAB*

Hit <Enter> to continue:>> RET

Press <Enter> to compare linear and circular convolution:>> RET



*ALT-TAB*

Hit <Enter> to continue:>> RET

Run C-Graph again? Enter 'y' or 'n'.

C-Graph:>> n

Exiting C-Graph ...

Bye.

On exit, the directory `c-graphs` will be created containing the 3 graphs from the last run:

- `signals.png`
- `transforms.png`, and
- `convolution.png`

These may be printed or displayed with Image Magick or another graphics editor of your choosing.



## 6 Reporting Bugs

To report bugs or suggest enhancements for C-Graph, please send electronic mail to [agt@codeartnow.com](mailto:agt@codeartnow.com).

For bug reports, please include enough information for the maintainers to reproduce the problem. Generally speaking, that means:

- The version numbers of C-Graph (which you can find by running ‘`c-graph --version`’) and any other program(s) or manual(s) involved.
- Hardware and operating system names and versions.
- The contents of any input files necessary to reproduce the bug.
- The expected behavior and/or output.
- A description of the problem and samples of any erroneous output.
- Options you gave to `configure` other than specifying installation directories.
- Anything else that you think would be helpful.

When in doubt whether something is needed or not, include it. It’s better to include too much than to leave out something important.

Patches are welcome. Please follow the existing coding style.



## Appendix A Sketch of Convolution Theory

### A.1 Introductory Ideas

C-Graph compares the linear and circular convolution of two signals. Subroutine `convo` computes the linear convolution directly in the time domain, while the FFT is exploited to compute circular convolution through the convolution theorem, which is defined below (see [Section A.2.2 \[The Convolution Theorem\]](#), page 28).

Convolution is an operation by which two functions combine to produce a third that represents a kind of moving average. This is a naturally occurring phenomenon that presents itself whenever there is a linear system obeying the principles of superposition and shift/time invariance. Accordingly, the mathematics of convolution has found application to much of science and engineering in areas ranging from statistics to computer vision.

The output of any linear shift invariant system may be described as the convolution of the input with the impulse response of the system. In computer vision, for example, where the system being considered is a 2-dimensional image, the output of the system may be blurred as a result of the relative motion of the camera and the object. This blurred image can be modelled by convolution of the static image with the 2-dimensional impulse response.

The 2-dimensional impulse response is called a *pointspread function* (PSF). Each pixel in the image produces a copy of the PSF, scaled according to the strength of the pixel and spatially shifted. Superposition of these copies form the resultant output signal, the system being linear and shift invariant. The output blurred image is then a convolution that is, in fact, a linear combination of the PSFs. The design of a filter for image restoration must then rely on inverse convolution.

A thorough treatment of the mathematics of convolution is beyond the scope of this manual. See [Appendix B \[Appendix B\]](#), page 35, for some references on the subject, and related engineering theory.

### A.2 The Mathematics

Consideration of the 1-dimensional case simplifies the arithmetic. To prove the convolution theorem, we first derive an expression for the convolution of 2 signals, then apply the Fourier transform to this expansion.

#### A.2.1 Deriving the Convolution Sum

A discrete-time signal may be modelled as a series of piecewise rectangular pulses. The summation of all such rectangular pulses approximates the signal  $f$ :

$$f(n) = \sum_m f(m) \text{rect}(n - m)$$

where  $n - m$  denotes the rectangle whose base on the  $n$  axis is centred at sample  $n = m$ .

In the limit, the series of rectangular pulses approaches a continuous signal as the pulse width tends to zero and each pulse becomes an impulse signal (figure ?). Each impulse signal can then be represented as a scaled and shifted unit impulse simulating one sample of the discrete signal.

$$f(n) = \sum_m f(m) \delta(n - m)$$

Applying a system transform  $M$  that maps the input signal  $f$  to the output signal  $g$ ,

$$\begin{aligned} g(n) &= M[f(n)] \\ g(n) &= M [f(n) = \sum_m f(m) \delta(n - m)] \\ &= \sum_m f(m) M[\delta(n - m)] \end{aligned}$$

Since the system transforms a delta function to the system *impulse response*  $h$

$$g(n) = \sum_m f(m) h(n - m) \quad (1)$$

The above expression called the *convolution sum*, denoted by  $f(*)h$ , defines the output  $g(n)$  of the system.

## A.2.2 The Convolution Theorem

C-Graph demonstrates the *convolution theorem*. The convolution of 2 signals in the time domain is equal to the inverse Fourier transform of the product of their transforms in the frequency domain.

Just as a signal can be represented by a linear combination of scaled and shifted impulses, we can also describe the signal as a linear combination of sinusoidal basis functions. The Fourier transform exploits this representation to deconstruct the signal into frequency components, each corresponding to a basis sinusoid.

Using Euler's identity

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

The sinusoidal sum representing the discrete time signal may be written in the form

$$f(n) = 1/N \sum_{k=0}^{N-1} F(k) e^{j\omega_k n}$$

where  $\omega_k = 2k\pi/N$ , and the  $F(k)$  are Fourier transform coefficients indicating the strength of the  $k$ th spectral sample of frequency  $\omega_k$  (how much of the each basis sinusoid is present in the signal).<sup>1</sup>

Accordingly, the  $F(k)$  may be computed from the signal

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{-j\omega_k n}$$

This is the Fourier transform description of the signal as a function of frequency.

From eqn (1), the convolution of an input signal  $f$  with the system impulse response  $h$  to give an output  $g$  is defined as:

<sup>1</sup> For a periodic signal, these coefficients are the  $\delta$  functions of the Fourier transform.



$$g(n) = f(*)h = \sum_m f(m) h(n - m)$$

Let the Fourier transform of  $g(n)$  be denoted by

$$\Gamma[g(n)] = \sum_n g(n) e^{-j\omega n}, \text{ then}$$

$$\Gamma[f(*)g] = \sum_n [f(*)g] e^{-j\omega n}$$

$$\begin{aligned} \Gamma[f(*)h] &= \sum_n \sum_m f(m) h(n - m) e^{-j\omega n} \\ &= \sum_m f(m) \sum_n h(n - m) e^{-j\omega n} \end{aligned}$$

Changing the variable to  $p = n - m$

$$\begin{aligned} \Gamma[g(n)] &= \sum_m f(m) \sum_p h(p) e^{-j\omega(m+p)} \\ &= \sum_m f(m) e^{-j\omega m} \sum_p h(p) e^{-j\omega p} \end{aligned}$$

Taking the inverse Fourier transform,

$$f(*)g = \Gamma^{-1}[\Gamma[f] \Gamma[h]]$$

This is the convolution theorem.

### A.3 Linear and Circular Convolution

The simulation of actual or linear convolution requires a sequence of multiplications and additions that are computationally too slow for high speed operations such as deblurring filters for precision vision control systems. The *FFT*, an algorithm for efficiently computing the DFT, dramatically overcomes the computational load by successively decomposing the multiplication of two sequences into subsequences of half the length thereby reducing the number of arithmetic operations by roughly  $N/\log N$ .

The cost of this additional computational power is the treatment of the convolving signals as periodic with  $N$  samples per period. The resulting convolution is termed *circular convolution*. It can be shown that circular convolution and linear convolution are equivalent if  $N \geq L + P - 1$  where  $L, M$  are the unpadding lengths of the sequences being convolved.

We illustrate the difference between linear and circular convolution using abbreviated sequences for the pulses demonstrated in [Chapter 5 \[A C-Graph Session\]](#), page 13

#### A.3.1 Linear Convolution

As noted above (see [Section A.1 \[Introductory Ideas\]](#), page 27), in linear convolution, each sample of  $f$  contributes a scaled and shifted copy of the  $h$ . This is accomplished by the multiplication of the particular sample  $m$  of  $f$  by each sample of  $h$ .

This sequential multiplication can be visualized as a physical reflection of  $h(m)$  about the vertical axis to obtain  $h(-m)$  followed by discrete shifts of 1 sample interval ( $\delta n = 1$ ) along the time axis with no overlap of the signals at the beginning and end of the translation. As the impulse response moves along the time axis, the point by point multiplication of

coincident samples is summed. The sum at each point in the translation is the value of the convolution sum  $g(n)$  at that point, and the length of the convolution is  $L + M$ .figure?

For the sequences

$$f(m) = [1, 1, 1], \text{ and}$$

$$h(m) = [0, 1, 2]$$

The series of operations for linear convolution

$$f(*)h = [0 \ 1 \ 3 \ 3 \ 2], \text{ are:}$$

$$\begin{array}{r}
 0. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(0) = f(m)h(0-m) \qquad 0+0+0+0+0 = 0
 \end{array}$$

$$\begin{array}{r}
 1. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(1) = f(m)h(1-m) \qquad 0+1+0+0 = 1
 \end{array}$$

$$\begin{array}{r}
 2. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(2) = f(m)h(2-m) = \qquad 2+1+0 = 3
 \end{array}$$

$$\begin{array}{r}
 3. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(3) = f(m)h(3-m) = \qquad 0+2+1+0 = 3
 \end{array}$$

$$\begin{array}{r}
 4. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad \qquad \qquad 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \qquad \qquad \text{-----} \\
 g(4) = f(m)h(4-m) = \qquad 0+0+2+0+0 = 2
 \end{array}$$

### A.3.2 Circular Convolution

We can imagine circular convolution in terms of the relative rotation of two concentric cylinders whose circumferences are of length  $N$ . A copy of the  $N$  samples comprising  $f$  is wrapped anticlockwise round one cylinder, while a copy of  $h$  is wrapped clockwise round the other cylinder, reflecting  $h$ . Rotating the second cylinder anticlockwise by 1 sample interval each time, multiplying the coincident samples and summing will give corresponding values of the convolved signal  $g$ .

$$\begin{aligned}
 f(m) &= [1, 1, 1], \text{ and} \\
 h(m) &= [0, 1, 2]
 \end{aligned}$$

The series of operations for circular convolution

$$\begin{array}{r}
 f(*)h = [0 \ 1 \ 3 \ 3 \ 2], \text{ are:} \\
 \\
 0. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad \qquad \qquad 0 \ 2 \ 1 \\
 \qquad \qquad \qquad \qquad \qquad \text{-----} \\
 g(0) = f(m)h(0-m) \qquad 0+2+1 = 3 \\
 \\
 1. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad \qquad \qquad 1 \ 0 \ 2 \\
 \qquad \qquad \qquad \qquad \qquad \text{-----} \\
 g(1) = f(m)h(1-m) \qquad 1+0+2 = 3
 \end{array}$$

$$\begin{array}{r}
 2. \qquad \qquad \qquad 1 \ 1 \ 1 \\
 \qquad \qquad \qquad 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(2) = f(m)h(2-m) \quad 2+1+0 = 3
 \end{array}$$

By zero-padding each sequence of length  $L = 3$  to length  $N$  so that  $N \geq L + L - 1$  (see [Section A.3 \[Linear and Circular Convolution\], page 29](#)), we obtain the sequences

$$\begin{aligned}
 f(m) &= [1, 1, 1, 0, 0] \\
 h(m) &= [0, 1, 2, 0, 0]
 \end{aligned}$$

Circular convolution then achieves the same as result as linear convolution:

$$f(*)h = [0 \ 1 \ 3 \ 3 \ 2]$$

The operations are:

$$\begin{array}{r}
 0. \qquad \qquad \qquad 1 \ 1 \ 1 \ 0 \ 0 \\
 \qquad \qquad \qquad 0 \ 0 \ 0 \ 2 \ 1 \\
 \qquad \qquad \qquad \text{-----} \\
 g(0) = f(m)h(0-m) \quad 0+0+0+2+1 = 0 \\
 \\
 1. \qquad \qquad \qquad 1 \ 1 \ 1 \ 0 \ 0 \\
 \qquad \qquad \qquad 1 \ 0 \ 0 \ 0 \ 2 \\
 \qquad \qquad \qquad \text{-----} \\
 g(1) = f(m)h(1-m) \quad 1+0+0+0+2 = 1 \\
 \\
 2. \qquad \qquad \qquad 1 \ 1 \ 1 \ 0 \ 0 \\
 \qquad \qquad \qquad 2 \ 1 \ 0 \ 0 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(2) = f(m)h(2-m) \quad 2+1+0+0+0 = 3
 \end{array}$$

$$\begin{array}{r}
 3. \qquad \qquad \qquad 1 \ 1 \ 1 \ 0 \ 0 \\
 \qquad \qquad \qquad 0 \ 2 \ 1 \ 0 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(3) = f(m)h(3-m) \quad 0+2+1+0+0 = 3
 \end{array}$$

$$\begin{array}{r}
 4. \qquad \qquad \qquad 1 \ 1 \ 1 \ 0 \ 0 \\
 \qquad \qquad \qquad 0 \ 0 \ 2 \ 1 \ 0 \\
 \qquad \qquad \qquad \text{-----} \\
 g(4) = f(m)h(4-m) \quad 0+0+2+1+0 = 2
 \end{array}$$



## Appendix B References

The sources below were consulted in the preparation of C-Graph and/or the 1983 dissertation [1] from which C-Graph is derived.

1. Thompson, Adrienne G. "Interactive Computer Package Demonstrating: Sampling Convolution and the FFT", BSc. Engineering Honours thesis, University of Aberdeen (Scotland), 1983, see <http://codeartnow.com/law-project>.
2. Horn, Berthold, K. P. *Robot Vision*. MIT Press, Cambridge, Massachusetts, 1986.
3. McGillem, Clare D., and Cooper, George R. *Continuous and Discrete Signal and System Analysis*. Holt, Rinehart and Winston, Inc., 1990.
4. Oppenheim, Alan V., and Schaffer, Ronald W. *Digital Signal Processing*. Prentice-Hall, Englewood Cliffs, N.J., 1975.
5. Smith, Steven W. *The Scientist and Engineer's Guide to Digital Signal Processing* <http://www.dspguide.com>
6. Stremmler, Ferrel G. *Introduction to Communication Systems*. Addison-Wesley Publishing Co. Inc, 1977.
7. Wouk, Arthur. *fft-simple.f90*. Ed. Alan Miller. 2003. Retrieved from [http://jblevins.org/mirror/amiller/fft\\_simple.f90](http://jblevins.org/mirror/amiller/fft_simple.f90)
8. Yuen, C. K. and Fraser, D. *Digital Spectral Analysis*. CSIRO/Pitman, East Melbourne, Australia, 1979.
9. Ziemer, Rodger E., Tranter, William H., Fannin, D. Ronald. *Signals Systems Continuous and Discrete*. 4th ed. Prentice Hall, Upper Saddle River, NJ 07458 1998.





## Appendix C Signals Gallery

Here are some examples of signal pairs from which you can make convolution art with C-Graph.



# Appendix D GNU Free Documentation License

Version 1.3, 3 November 2008

Copyright © 2000, 2001, 2002, 2007, 2008 Free Software Foundation, Inc.

<http://fsf.org/>

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

## 0. PREAMBLE

The purpose of this License is to make a manual, textbook, or other functional and useful document *free* in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or non-commercially. Secondly, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of “copyleft”, which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

## 1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The “Document”, below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as “you”. You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A “Modified Version” of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A “Secondary Section” is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document’s overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The “Invariant Sections” are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released

under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The “Cover Texts” are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A “Transparent” copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not “Transparent” is called “Opaque”.

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The “Title Page” means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, “Title Page” means the text near the most prominent appearance of the work’s title, preceding the beginning of the body of the text.

The “publisher” means any person or entity that distributes copies of the Document to the public.

A section “Entitled XYZ” means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as “Acknowledgements”, “Dedications”, “Endorsements”, or “History”.) To “Preserve the Title” of such a section when you modify the Document means that it remains a section “Entitled XYZ” according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

## 2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

### 3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

### 4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections 2 and 3 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

- A. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any,

- be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.
- B. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.
  - C. State on the Title page the name of the publisher of the Modified Version, as the publisher.
  - D. Preserve all the copyright notices of the Document.
  - E. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.
  - F. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.
  - G. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.
  - H. Include an unaltered copy of this License.
  - I. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.
  - J. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.
  - K. For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.
  - L. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.
  - M. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.
  - N. Do not retitle any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.
  - O. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their

titles to the list of Invariant Sections in the Modified Version’s license notice. These titles must be distinct from any other section titles.

You may add a section Entitled “Endorsements”, provided it contains nothing but endorsements of your Modified Version by various parties—for example, statements of peer review or that the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

## 5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled “History” in the various original documents, forming one section Entitled “History”; likewise combine any sections Entitled “Acknowledgements”, and any sections Entitled “Dedications”. You must delete all sections Entitled “Endorsements.”

## 6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

## 7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an “aggregate” if the copyright resulting from the compilation is not used to limit the legal rights of the compilation’s users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document’s Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

## 8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled “Acknowledgements”, “Dedications”, or “History”, the requirement (section 4) to Preserve its Title (section 1) will typically require changing the actual title.

## 9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided under this License. Any attempt otherwise to copy, modify, sublicense, or distribute it is void, and will automatically terminate your rights under this License.

However, if you cease all violation of this License, then your license from a particular copyright holder is reinstated (a) provisionally, unless and until the copyright holder explicitly and finally terminates your license, and (b) permanently, if the copyright holder fails to notify you of the violation by some reasonable means prior to 60 days after the cessation.

Moreover, your license from a particular copyright holder is reinstated permanently if the copyright holder notifies you of the violation by some reasonable means, this is the first time you have received notice of violation of this License (for any work) from that copyright holder, and you cure the violation prior to 30 days after your receipt of the notice.

Termination of your rights under this section does not terminate the licenses of parties who have received copies or rights from you under this License. If your rights have been terminated and not permanently reinstated, receipt of a copy of some or all of the same material does not give you any rights to use it.



## 10. FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See <http://www.gnu.org/copyleft/>.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License “or any later version” applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation. If the Document specifies that a proxy can decide which future versions of this License can be used, that proxy’s public statement of acceptance of a version permanently authorizes you to choose that version for the Document.

## 11. RELICENSING

“Massive Multiauthor Collaboration Site” (or “MMC Site”) means any World Wide Web server that publishes copyrightable works and also provides prominent facilities for anybody to edit those works. A public wiki that anybody can edit is an example of such a server. A “Massive Multiauthor Collaboration” (or “MMC”) contained in the site means any set of copyrightable works thus published on the MMC site.

“CC-BY-SA” means the Creative Commons Attribution-Share Alike 3.0 license published by Creative Commons Corporation, a not-for-profit corporation with a principal place of business in San Francisco, California, as well as future copyleft versions of that license published by that same organization.

“Incorporate” means to publish or republish a Document, in whole or in part, as part of another Document.

An MMC is “eligible for relicensing” if it is licensed under this License, and if all works that were first published under this License somewhere other than this MMC, and subsequently incorporated in whole or in part into the MMC, (1) had no cover texts or invariant sections, and (2) were thus incorporated prior to November 1, 2008.

The operator of an MMC Site may republish an MMC contained in the site under CC-BY-SA on the same site at any time before August 1, 2009, provided the MMC is eligible for relicensing.

## ADDENDUM: How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

```
Copyright (C)  year  your name.
Permission is granted to copy, distribute and/or modify this document
under the terms of the GNU Free Documentation License, Version 1.3
or any later version published by the Free Software Foundation;
with no Invariant Sections, no Front-Cover Texts, and no Back-Cover
Texts. A copy of the license is included in the section entitled ‘‘GNU
Free Documentation License’’.
```

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the “with...Texts.” line with this:

```
with the Invariant Sections being list their titles, with
the Front-Cover Texts being list, and with the Back-Cover Texts
being list.
```

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.

# Index

- 
- dedicate ..... 7
- help ..... 7
- version ..... 7
- d ..... 7
- h ..... 7
- v ..... 7
  
- A**
- aperiodic signals ..... 9
  
- B**
- bugs, reporting ..... 25
  
- C**
- checklist for bug reports ..... 25
- circular convolution ..... 27, 31
- computer vision ..... 27
- convolution ..... 3, 5, 27, 28
- convolution sum ..... 27
- convolution theorem ..... 27, 28
- convolution, circular ..... 29
- convolution, linear ..... 29
  
- D**
- default values ..... 9
- delta function ..... 27
- DFT ..... 5, 29
- discrete Fourier transform, see DFT ..... 5
- Dissertation ..... 3, 35
  
- E**
- erroneous data ..... 9
- Euler's identity ..... 28
- example run ..... 13
  
- F**
- Fast Fourier Transform, see FFT ..... 5
- FFT ..... 5, 27, 29
- Fourier coefficient ..... 28
- Fourier transform ..... 28
- frequency ..... 10
- frequency domain ..... 28
- frequency, selecting ..... 9
  
- G**
- graphs, printing ..... 23
  
- I**
- impulse response ..... 27
- impulse signal ..... 27
- inverse Fourier transform ..... 28
  
- L**
- linear convolution ..... 27, 29
  
- N**
- number of samples ..... 9
  
- P**
- parameters ..... 9
- patches, contributing ..... 25
- periodic signals ..... 9
- periodic waveform, selecting ..... 9
- point spread function ..... 27
- printing graphs ..... 23
- pulse signals ..... 9
- pulses, selecting ..... 9
  
- S**
- samples, number of ..... 9
- sampling rate ..... 10
- scaling the signals ..... 9
- signals, types of waveforms ..... 9
- subroutine `conv` ..... 27
- system impulse response ..... 28
  
- T**
- Thesis ..... 35
- Thesis, see Dissertation ..... 3
- time domain ..... 28
  
- W**
- waveform parameters ..... 9

